NON-REVERSED SPECTRA OF RESTRICTED COINCIDENCE

By Carl Barus
DEPARTMENT OF PHYSICS, BROWN UNIVERSITY
Received by the Academy, September 15, 1916

In figure 1 the white ray L from the collimator is diffracted by the grating G and the two spectra a and a' thereafter reflected by the parallel opaque mirrors M and N, to be again diffracted by the grating G'. The rays are observed by a telescope at T. If the gratings, G, G' have the same constant, it is obvious that the field of the telescope will show a sharp white image of the slit, for each mirror. If M N G G' are adjusted for symmetry by aid of the adjustment screws on each and the rulings are parallel, the two white slit images will coincide horizontally and vertically. If now a direct vision spectroscopic prism, or a direct vision prism grating G'' is placed in front of the telescope, the superposed white slit images will be drawn out into overlapping non-reversed spectra, which will usually show a broad strip of interference fringes.[1]

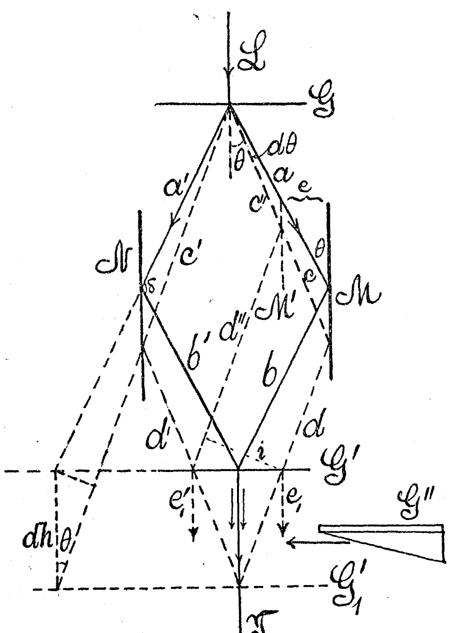

FIG. 1.

When first found, the fringes were very fine parallel lines, filling an irregular strip in the orange-yellow region, and it was already obvious that an enormous play of the micrometer screw at M would be permissible.

A number of film gratings were tested and the best samples selected ($D = 175 \times 10^{-6}$), although the dispersion was still too large and the D lines not clear. To secure more light, a beam of sunlight 15 cm. in diameter was condensed to a focus on the slit by a large lens of about 2 meters in focal distance. The illumination was now adequate and the fringes were found at once. These fringes, in view of more accurate adjustment, were also larger than before.

Figure 1 shows, if ab, $a'b'$ and cd, $c'd'$ are pairs of corresponding rays

of the same order but different wave lengths, λ and λ', respectively, that for the given position of G and G', only the rays a a' issue coincidently at T. The rays cd, $c'd'$ issue at e_1, e'_1 and though brought to the identical focus by the telescope, the distance e_1 e'_1 may be too large to admit of appreciable interference. Hence the colored strip within which interferences occur will comprise those wave lengths which lie very near λ, whereas the colors lying near λ', etc., will be free from interference.

If the mirror M is displaced parallel to itself to M' by the micrometer screw, the rays c'' d'', and c' d' will now coincide at e'_1, whereas the rays from a b and $a'b'$ will no longer issue coincidently and may not interfere. Thus the interferences are transferred as a group from rays lying near λ to rays lying near λ'. It is obvious therefore that with the displacement of M, the strip carrying interferences will shift through the spectrum and that a relatively enormous play of the micrometer slide at M will be available without the loss of interferences. In fact a displacement, e, of over 3 cm. of M normal to itself, produced no appreciable change in the size or form of fringes, but they passed from the green region into the red. The fringes as seen with a fine slit were straight parallel strong lines. They did not thin out to hair lines at their ends, nor show curvature, as one would be inclined to anticipate. On the contrary, they terminated rather abruptly at the edges of a strip occupying about one-fourth of the visible length of the spectrum.

It follows, from figure 1, that the displacement of M does not change the lengths of rays; for they are enclosed between parallel planes, as it were. Since the double angle of reflection is here $\delta = 180° - 2\theta$, where θ is the angle of diffraction of G and G', the displacements of M over a normal distance e will shorten the path at M in accordance with the equation

$$n\,\lambda = 2e \cos \delta/2 = 2e \sin \theta \qquad (1)$$

where n is the number of fringes passing at wave length λ.

This equation[2] is not obvious, as for constant λ, the distance between G and G' measured along a given ray, for any position of M or N, is also constant. The equation may be corroborated by drawing the diffracted wave front i for instance, which cuts off a length $2e \sin \theta$ from d''.

Since $\sin \theta = \lambda/D$ if D is the grating space, the last equation becomes

$$n = 2e/D$$

or per fringe

$$\delta e = D/2,$$

a remarkable result, showing that the displacement of the mirror M per fringe is independent of wave length and equal to half the grating space. An interferometer independent of λ and available throughout relatively enormous ranges of displacement is thus at hand. It appears that it is also independent of the angle of incidence at G.

In case of the given grating and sodium light $\theta = 19° 37'$. Hence if δe is the displacement per fringe[2]

$$e = \lambda/2 \sin \theta = 10^{-6} \times 88 \text{ cm.}$$

Actuating the micrometer at M directly by hand the following rough data were successively obtained from displacements corresponding to 10 fringes:

$$10^6 \times \delta e = 65 \quad 95 \quad 90 \quad 80 \quad 60 \text{ cm.}$$

Without special precaution the fine fringes can not be counted closer than this, so that the data are corroborative.

The fore and aft motion of G' produces no separate shift of fringes while the fringe bearing strip is displaced as a whole in mean wave length. Figure 1 shows at once that if G' were displaced to G'_1, the λ rays $b\, b'$ would lose their coincidence in T, while that property would now be possessed by the λ' rays $d\, d'$. But the same path difference is added to both d and d'. The ratio of corresponding displacements at M and G' is $\tan \theta : 1$. Equation (1) is of interest in interferometry, in view of the very long ranges of displacements available. For such purposes gratings of lower dispersion (preferably ruled gratings or else prisms) may be used to obtain greater luminous intensity in the spectrum. Path difference may also be introduced by compensators. If a thin sheet of mica is moved in either b or b' there is a lively skirmish of fringes, but they do not change size appreciably. A plate 2.8 millimeters thick with strong fringes horizontal in the yellow, if placed in the b' pencil produces hair lines inclined toward the left in the red; if placed in the b pencil, hair lines inclined to the left in the green; etc. Plates were tested up to 2 cm. To fully exhibit their effect it is necessary to produce the elliptic fringes, presently to be referred to. The shift from red to green, if produced without compensators by the displacement of M, shows scarcely any variation of fringes, either as to size or inclination.

To change the size of fringes it is easiest to rotate the grating G' (relatively to G) on a horizontal axis normal to itself. They then both rotate and grow coarser, usually attaining the maximum of size when the fringes are vertical. Fringes quite large and black may be obtained

in this way, which are naturally much more sensitive. Size may also be changed by compensators and this method is usually more available.

In addition to the above experiments, work was done at some length with homogeneous light, with gratings of different constants, etc., which cannot be detailed here. The most interesting result was obtained with a *wide* slit and white light. It was shown that the fringes are ultimately nearly confocal ellipses of enormous eccentricity and with the major axis vertical. To produce and center them, the refraction (dispersion) of plates is advantageous, if not necessary.

It is now of interest to turn to the displacement of G', normal to itself, and to consider the resolving power of the system. For the latter bears a close analogy to the experiments made in a preceding paper. (Carnegie Publication, No. 249, § 37 et. seq., 1916) on the behavior of crossed rays. If G' is displaced to G'_1, over a distance $e' = dh$ (see figure 1 where h is the distance apart of G and G'), the rays λ' meeting in T will now be in the same condition as were originally the rays λ. In other words, e'_1 and e'_1 have become coincident at G'. If we assume that rotationally the same type of fringe results in these cases, and if $\lambda' - \lambda = d\lambda$, $\theta - \theta' = d\theta$, (for the passage of bb' into dd' is in the direction from red to violet)

$$d\,\theta = dh \sin\theta \cos\theta/h, \qquad \text{nearly.}$$

Since $\lambda = D \sin\theta$ and $d\,\lambda = -D\cos\theta\, d\,\theta$, this may be changed to

$$d\,\lambda/\lambda = dh\,(1 - \lambda^2/D^2)/h$$

when D is the grating constant.

The present method, apart from any practical outcome, is worth pursuing because of the data it will furnish of the width of the strip of spectrum carrying interference fringes, under any given conditions. For here the spectra are not reversed or inverted and the latitude of interference or diffraction throughout λ is much broader than in case of reversed spectra. But for this purpose films will not suffice and rigid refracting systems must be devised, and the grating constants must be quite identical.

[1] This article is a note from a Report to the Carnegie Institution of Washington.

[2] The equation is also true for oblique incidence. But for this and the use of homogeneous light with a wide slit, the availability of gratings of different constants, etc., the report mentioned may be consulted.